一元二次方程式 想法源頭：CA 設計者：陳梅仙 修改：余秉橙

1.**複習舊經驗**！

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| --- | --- |
| **一起算** ⇨ **分開算** | **分開算** ⇨ **一起算** |
| (1) (x-5)(x-5) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_   |  |  |  | | --- | --- | --- | |  | x | -5 | | ×) | x | -5 | |  | □ | □ | | □ | □ |  | | □ | □ | □ | | (2) =(\_\_\_\_)×(\_\_\_\_)=(\_\_\_\_)2   |  |  |  | | --- | --- | --- | |  | □x | □ | | ×) | □x | □ | |  | □x | □ | | □ | □x |  | |  |  |  | |
| (3) (x+7)(x-7) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_   |  |  |  | | --- | --- | --- | |  | x | +7 | | ×) | x | -7 | |  | □ | □ | | □ | □ |  | | □ | □ | □ | | (4) =(\_\_\_\_)×(\_\_\_\_)=(\_\_\_\_)2   |  |  |  | | --- | --- | --- | |  | □x | □ | | ×) | □x | □ | |  | □x | □ | | □ | □x |  | |  |  |  | |
| (5) (x+4)(x-6) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_   |  |  |  | | --- | --- | --- | |  | x | +4 | | ×) | x | -6 | |  | □ | □ | | □ | □ |  | | □ | □ | □ | | (6) =(\_\_\_\_)×(\_\_\_\_)   |  |  |  | | --- | --- | --- | |  | □x | □ | | ×) | □x | □ | |  | □x | □ | | □ | □x |  | |  |  |  | |
| (7) (3x+4)(2x-7) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_   |  |  |  | | --- | --- | --- | |  | 3x | +4 | | ×) | 2x | -7 | |  | □ | □ | | □ | □ |  | | □ | □ | □ | | (8) =(\_\_\_\_)×(\_\_\_\_)   |  |  |  | | --- | --- | --- | |  | □x | □ | | ×) | □x | □ | |  | □x | □ | | □ | □x |  | |  |  |  | |

2. **解一元二次方程式的兩個想法**

**(一)配方法**(透過搭配一個數字來配成完全平方)

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| |  |  |  | | --- | --- | --- | |  | □x | □ | | ×) | □x | □ | |  | □x | □ | | □x2 | □x |  | | □x2 | □x | □ | | (1) x2+12x+\_\_\_\_\_\_=(\_\_\_\_\_\_\_)2  (2) x2-8x+\_\_\_\_\_\_\_=(\_\_\_\_\_\_\_)2  (3) x2+14x+\_\_\_\_\_\_=(\_\_\_\_\_\_\_)2  (4) x2-2x+\_\_\_\_\_\_\_=(\_\_\_\_\_\_\_)2 | (5) x2+7x+\_\_\_\_\_\_\_=(\_\_\_\_\_\_\_)2  (6) x2-11x+\_\_\_\_\_\_=(\_\_\_\_\_\_\_)2  (7) 2x2+6x+\_\_\_\_\_\_= 2(\_\_\_\_\_\_\_)2  (8) 3x2-12x+\_\_\_\_\_\_=\_\_(\_\_\_\_\_\_\_)2 |

**(二)因式分解法**(將分開算的算式轉化為合併算的算式)

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| |  |  |  | | --- | --- | --- | |  | □x | □ | | ×) | □x | □ | |  | □x | □ | | □x2 | □x |  | | □x2 | □x | □ | | (1)x2+7x-18=(\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_)  (2)x2－x－6=(\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_)  (3)6x2＋17x＋5=(\_\_\_\_\_)(\_\_\_\_\_)  (4)4x2＋11x－3=(\_\_\_\_\_)(\_\_\_\_\_) | (5)6x2－x－12=(\_\_\_\_\_\_)(\_\_\_\_\_\_)  (6)3x2－7x＋2=(\_\_\_\_\_\_)(\_\_\_\_\_\_)  (7)12x2－8x－15=(\_\_\_\_\_)(\_\_\_\_\_)  (8)6x2-5x-6=(\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_) |

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| |  |  |  | | --- | --- | --- | |  | x | □ | | ×) | x | □ | |  | □x | □ | | x2 | □x |  | | x2 | -5x | □ | | (9)x2-5x+\_\_\_\_\_\_\_=(\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_)  (10)x2-5x+\_\_\_\_\_\_\_=(\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_)  (11)x2-5x+\_\_\_\_\_\_\_=(\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_)  (12)x2-5x+\_\_\_\_\_\_\_=(\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_) |

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| |  |  |  | | --- | --- | --- | |  | x | □ | | ×) | x | □ | |  | □x | □ | | x2 | □x |  | | x2 | □x | -12 | | (13)x2+\_\_\_\_\_\_\_x-12=(\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_)  (14)x2+\_\_\_\_\_\_\_x-12=(\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_)  (15)x2+\_\_\_\_\_\_\_x-12=(\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_)  (16)x2+\_\_\_\_\_\_\_x-12=(\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_) |

3.利用一條長度為20的繩子圍出長方形，繩子的長度要剛好用完

(1)請完成下列表格

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 長 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | x |
| 寬 |  |  |  |  |  |  |  |  |  |  |  |  |
| 面積 |  |  |  |  |  |  |  |  |  |  |  |  |

(2)什麼情況下可以圍出最大的長方形面積呢？ (3)什麼情況下可以圍出面積為22的長方形呢？

4.請寫等式中□和△可能的答案，感覺一下這些等式中所傳達的□,△值的確定性和不確性。

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| ❶□×△＝6 |  | ❷□×△＝1 |  | ❸□×△＝0 |  |

5.請試著找出方程式的解

(1) (x+1)(x-2) =0 (2) (x-2)(x-1) =2 可以解嗎？

6.解一元二次方程式

一、**準備**

|  |  |
| --- | --- |
| 1.**求平方根** 、 | 2.**兩數相乘等於零** |
| (1)求方程式x2=5的解  x2=5  ⇨ ( x )×( x )=5  ⇨ ( )×( )=5  ⇨ x=( ) | (1)求方程式(x-2)(x-5)=0的解  (x-2)(x-5)=0  ⇨ ( )=0或 ( )=0  ⇨ x=( ) |
| (2)求方程式x2=-5的解  x2=-5  ⇨ ( x )×( x )=-5  ⇨ ( )×( )=-5  ⇨ x=( ) | (2)求方程式(x)(2x-7)=0的解  (x)(2x-7)=0  ⇨ ( )=0或 ( )=0  ⇨ x=( ) |
| (3)求方程式(x-3)2=5的解  (x-3)2=5  ⇨ ( x-3 )×( x-3 )=5  ⇨ ( )×( )=5  ⇨ x=( ) | (3)求方程式x2+4x=0的解  x2+4x=0  ⇨ ( x )×( )=0  ⇨ ( )=0或 ( )=0  ⇨ x=( ) |
| (4)求方程式2(x+3)2=5的解  2(x+3)2=5  ⇨ ( x+3 )×( x+3 )=  ⇨ ( )×( )=  ⇨ x=( ) | (4)求方程式3x2-5x=0的解  3x2-5x=0  ⇨ ( x )×( )=0  ⇨ ( )=0或 ( )=0  ⇨ x=( ) |

二、**執行**

|  |  |
| --- | --- |
| 配成平方法(**一定可以**) | 因式分解法(**有時不行**) |
| (1)求方程式x2+8x+7=0的解  x2+8x+7=0  ⇨ x2+8x=( )  ⇨ x2+8x+□=( )+□  ⇨( ) 2=( )  ⇨x=( )   |  |  |  | | --- | --- | --- | |  | □x | □ | | ×) | □x | □ | |  | □x | □ | | x2 | □x |  | | x2 | +8x | □ | | (1)求方程式x2+8x+7=0的解  x2+8x+7=0  ⇨ ( )×( )=0  ⇨ ( )=0或 ( )=0  ⇨ x=( )   |  |  |  | | --- | --- | --- | |  | □x | □ | | ×) | □x | □ | |  | □x | □ | | x2 | □x |  | | x2 | +8x | +7 | |
| (2)求方程式x2-10x+21=0的解  x2-10x+21=0  ⇨ x2-10x=( )  ⇨ x2-10x+□=( )+□  ⇨( ) 2=( )  ⇨x=( )   |  |  |  | | --- | --- | --- | |  | □x | □ | | ×) | □x | □ | |  | □x | □ | | x2 | □x |  | | x2 | -10x | □ | | (2)求方程式x2-10x+21=0的解  x2-10x+21=0  ⇨ ( )×( )=0  ⇨ ( )=0或 ( )=0  ⇨ x=( )   |  |  |  | | --- | --- | --- | |  | □x | □ | | ×) | □x | □ | |  | □x | □ | | x2 | □x |  | | x2 | -10x | +21 | |
| (3)求方程式2x2+3x-5=0的解  2x2+3x-5=0  ⇨ 2x2+3x =( )  ⇨ x2+x+□=( )+□  ⇨( ) 2=( )  ⇨x=( )   |  |  |  | | --- | --- | --- | |  | □x | □ | | ×) | □x | □ | |  | □x | □ | | x2 | □x |  | | x2 | +x | □ | | (3)求方程式2x2+3x-5=0的解  2x2+3x-5=0  ⇨ ( )×( )=0  ⇨ ( )=0或 ( )=0  ⇨ x=( )   |  |  |  | | --- | --- | --- | |  | □x | □ | | ×) | □x | □ | |  | □x | □ | | 2x2 | □x |  | | 2x2 | +3x | -5 | |

公式解法：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 求方程式ax2+bx+c=0的解，a≠0  ax2+bx+c=0  ⇨ ax2+bx=( )  ⇨ x2+( )=( )  ⇨ x2+( )+□=( )+□  ⇨( ) 2=  (1) □ 0 ⇨ x=無解  (2) □ 0 ⇨ x=(重根)  (3) □ 0 ⇨ x= | |  |  |  | | --- | --- | --- | |  | □x | □ | | ×) | □x | □ | |  | □x | □ | | x2 | □x |  | | x2 | +x | □ | |

請利用**公式解**計算下面方程式的解

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| (1) x2+6x+8=0  a=( ) b=( ) c=( )  =( )  ⇨□0 (填入>.<.=)  ⇨ x=( ) | (2) x2+6x+9=0  a=( ) b=( ) c=( )  =( )  ⇨□0 (填入>.<.=)  ⇨ x=( ) | (3) x2+6x+10=0  a=( ) b=( ) c=( )  =( )  ⇨□0 (填入>.<.=)  ⇨ x=( ) |
| (4) x2+6x-8=0  a=( ) b=( ) c=( )  =( )  ⇨□0 (填入>.<.=)  ⇨ x=( ) | (5) x2+6x-9=0  a=( ) b=( ) c=( )  =( )  ⇨□0 (填入>.<.=)  ⇨ x=( ) | (6) x2+6x-10=0  a=( ) b=( ) c=( )  =( )  ⇨□0 (填入>.<.=)  ⇨ x=( ) |
| (7) 9x2-11x+4=0  a=( ) b=( ) c=( )  =( )  ⇨□0 (填入>.<.=)  ⇨ x=( ) | (8) 9x2-12x+4=0  a=( ) b=( ) c=( )  =( )  ⇨□0 (填入>.<.=)  ⇨ x=( ) | (9) 9x2-13x+4=0  a=( ) b=( ) c=( )  =( )  ⇨□0 (填入>.<.=)  ⇨ x=( ) |

6.解一元二次方程式part1：**因式分解法**

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| --- | --- |
| 1. x2-36=0   x2  ×  -36 | 1. 4x2-25=0   4x2  -25  × |
| 1. x2+6x+5=0   -5  +5  +6x  x2  × | 1. x2-2x-15=0   -15  -2x  x2  × |
| 1. 6x2+13x-5=0   +13x  6x2  × | 1. 12x2+7x-10=0   12x2  ×  -10  +7x |
| 1. 3x2+5x=12   +5x  3x2  × | 1. 10x2-13x=3   -13x  10x2  × |
| 1. (2x+1)2-4(2x+1) -21=0   ×  × | 1. 6(x-1)2+17(x-1)+5=0 |

7.解一元二次方程式part2：**配成平方法**

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| 1. x2=16 | 1. x2-5=0 |
| 1. (x-5)2=9 | 1. 3(x+2)2-21=0 |
| 1. x2+6x=5   +6x  x2  × | 1. x2-10x=-8   ×  -10x  x2 |
| 1. x2+14x+5=0   +14x  x2  × | 1. x2-2x+3=0   ×  -2x  x2 |
| 1. 2x2+6x=7   × | 1. 3x2-12x=4   × |

加強演練

|  |  |
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| 1. 解下列各一元二次方程式： (1) 2*x*2－*ax*－2*x*＋*a*＝0 (2) *x*2－*ax*＋*bx*－*ab*＝0 | 2. 甲、乙二生同解一個*x*2項係數是1的二次方程式，甲將*x*項的係數看錯，求得兩根為3與－6；乙將常數項看錯，求得兩根為3和4，若除此之外無其他的計算錯誤，則：(1) 正確的方程式為何？(2) 正確的兩根為何？ |

3. 求一元二次方程式(2*x*＋3)2－4(*x*－1)(2*x*＋3)＝12(*x*－1)2的解為。

4.已知一元二次方程式*x*2＋*ax*－16＝0的兩根均為整數，若*a*＞0且*a*為二位數，則*a*的個位數字與十位數字相差 。

5. 若3*x*2＋5*x*－8＝0 與6*x*2＋*ax*＋*b*＝0有相同的解，則2*a*－*b*＝？

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| 6.若*x*的二次方程式(*k*＋1)*x*2＋2*kx*＋(*k*2＋5*k*＋4)＝0，恰有一根為0，則*k*＝?，另一根＝?  8已知*xy*＜0，且3*x*2＋7*xy*－6*y*2＝0，則 ＝? | 7.解｜*x*2－3*x*－10｜＋｜*x*2＋5*x*＋6｜＝0，可得*x*＝?  9解 ＋＝，可得*x*＝? |

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| 10.(1) 利用公式解解方程式4*y*2＋5*y*－2＝0。 (2) 利用(1)題的結果，解方程式 (2) 4(5*x*－3)2＋5(5*x*－3)－2＝0。  13.  12. | | 11.已知*a*、*b*為方程式(*x*－5)2＝850的兩根，且 *a*＞*b*，利用下表，求*a*－*b*的近似值為何？   |  |  |  | | --- | --- | --- | | *N* |  |  | | 2 | 1.414 | 4.472 | | 3 | 1.732 | 5.477 | | 17 | 4.123 | 13.038 | | 85 | 9.220 | 29.155 | | |  |
| (1) 已知一元二次方程式*ax*2＋*bx*＋*c*＝0的兩根為α、β，請說明α＋β＝－、αβ＝。  (2) 若方程式*x*2＋(*k*＋7)*x*＋*k*－6＝0的兩根互為相反數，利用(1)題的結果，求*k*＝？ | (1) 已知α、β為方程式2*x*2＋*x*－6＝0的兩根，求(α＋4 )(β＋4 )＝？  (2) 若方程式2*x*2＋5*x*＋*k*－3＝0的兩根互為倒數，則*k*＝？ | |

14. (1) 將9*x*2－12*x*＋1化簡成(*ax*－*b*)2＋*c*的形式，若*a*＞0，則*a*、*b*、*c*之值為何？  
(2) 若4*x*2－(*m*－1)*x*＋9為完全平方式，則*m*＝?

(3)若3*x*2－18*x*＋11＝*a*(*x*＋*b*)2－*c*，則 ＝?

15.若*x*＝，則(3*x*＋1)2－9＝

16.若(*x*＋*m*)(*x*－3)＋4＝0的兩根相等，則*m*＝?

17.已知一線段長為5，其一端點坐標為(2 , 4)，而另一端點在*x*軸上，則此端點的坐標為?

|  |  |
| --- | --- |
| 18.若*x*2－5*x*＋1＝0，試求： (1) *x*＋＝? (2) *x*2＋＝? | 19.設α、β為*x*2－*px*＋＝0的兩根，且α＞β，則α－β＝? |

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| --- | --- |
| 20. *x*的二次方程式*x*(*x*－1)－*mx*＋(*m*＋1)＝0的兩根相等，則*m*＝? | 21.設*x*＝α為方程式*x*2＋3*x*－1＝0的一個根，則  (α＋5)(α＋2)(α＋1)(α－2)＝? |
| 22.已知*x*2＋*bx*＋*c*＝0可配方為(*x*＋5)2＝*m*，且*x*2＋(*b*＋8)*x*＋*c*＝0可配方為(*x*＋*k*)2＝*n*，則*k*＝? | 23.設*a*、*b*、*c*表示△*ABC*的三個邊長，若(*a*＋*c*)*x*2＋2*bx*＋(*a*－*c*)＝0有等根，則*a*、*b*、*c*何者為最長邊？ |

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| 24.在一片正方形金屬片的四個角落各剪去一個邊長為8公分的正方形，如圖(一)所示，然後摺成一個開口的置物盒，如圖(二)所示。如果這個置物盒的容積為1568立方公分，則這片金屬片原來的邊長為多少公分？ | | 25.圖(一)中，甲為邊長1公分的正方形，乙為一面積與甲相等的長方形，今發現若將甲放在乙上面，使它們兩鄰邊對齊，如圖(二)所示，結果所形成的區塊丙為正方形，則丙的邊長為何？ |
| 26.某雜貨店買進一籃20公斤的雞蛋，其中有  4公斤在搬運時不小心打破了，剩下的雞蛋每公斤以高於成本價12元的售價賣出。結算後發現，賺得的錢恰好是每公斤成本價的平方，則一籃雞蛋的成本價是每公斤多少元？  4-1128. 某基金因全球經濟不佳導致其淨值縮水，已知連續兩季結算，每季均比上一季縮水*x* %，原本25元的淨值，兩季後僅剩16元，則*x*＝? | 27. 成本200元的玩具，每個售價300元，每週可賣出100個。根據研究報告，得知售價每個多5元，則每週賣出的數量將少1個。若希望下週的利潤能剛好達到16000元，則售價應定為多少元？  29. 如圖，一個黃金矩形的長為*a*＋*b*，寬為*a*，  且已知(*a*＋*b*)：*a*＝*a*：*b*，則＝? | |

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| 30.如圖，有一個邊長為80公尺的正方形田地，若於內部開闢兩條交叉道路，使其分割出四塊全等的等腰直角三角形小田地，已知道路總面積為1500平方公尺，則道路寬為多少公尺。  4-06 | 31.有一個圓形跑道，若甲、乙兩人各自維持均速，甲跑一圈需費時10分鐘，現兩人同時同地反向而跑，乙在與甲相遇後，經過9分鐘回到原出發地，則乙跑一圈需費時多少分鐘。 |

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| 32.某件工程若由甲獨立完成，會比由乙獨立完成多10天。今由乙先獨作此工程5天後，改為甲和乙一起合作，需再9天才能完成此工程，則甲獨立完成此工程需多少天？ | 33.一輛工程車的後輪周長比前輪多2公尺，且每公里前輪比後輪多轉25圈，求後輪周長為多少公尺？ |

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| 34.已知*a*≠*b*，解方程式(*a*－*b*)*x*2＋(*b*－*c*)*x*＋(*c*－*a*)＝0，可得*x*＝? | 35.解22*x*－3＝3•2*x*－16，可得*x*＝? |

36.設*a*≠0，且*b*2－*ac*＞0，試將*ax*2－2*bx*＋*c*＝0利用配方法求出*x*＝?